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AXIAL CURRENTS, SUPERCURRENTS AND ANOMALIES IN SUPERSYMMETRIC QED

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The currents associated with the superconformal symmetries are defined as moments of the supercurrent, V_μ. All of the current (non-) conservation equations are known once the generalized trace of the supercurrent, $D^\alpha V_{\alpha\dot{\alpha}}$, is found. The superconformal anomalies are shown to have coefficients given by β of the Callan-Symanzik equation. In super QED there is an additonal U(1) axial current whose anomaly has a coefficient with no radiative corrections.

INTRODUCTION

In 1974 Ferrara and Zumino (1) discovered the existence of a supercurrent whose components contained the R-symmetry (chiral) current, the restricted supersymmetry current and the energy-momentum tensor. A further clarification of the relationships between these component currents and their various (non-) conservation equations was desired. In particular the Adler-Bardeen theorem (2) predicts that the coefficient of the anomaly for a suitably defined axial current has no radiative corrections. While on the other hand the scaling anomalies given by the trace of the energy-momentum tensor have a coefficient β of the Callan-Symanzik equation. However if the various currents as well as their anomalies are to be related by supersymmetry the coefficients of the anomalies should be the same (up to numerical factors).

In references (3) and (4) the supercurrent and axial current were studied in supersymmetric QED. It was shown that all of the superconformal currents are given by moments of the supercurrent, $V_{\alpha\dot{\alpha}}$, and all (non-) conservation equations of these currents were known once the generalized trace, $D^\alpha V_{\alpha\dot{\alpha}}$, of the supercurrent was found. This is completely analogous to the ordinary field theoretic case where all of the conformal currents are given by moments of the energy-momentum tensor, $T_{\mu\nu}$, and hence all conformal anomolies were given by its trace, T^λ_λ. The generalized trace was then found to have an anomaly whose coefficient was given by β of the Callan-Symanzik equation. In particular the R-symmetry axial current defined so that it was a component of the supercurrent had an anomaly coefficient of β. In super QED there was an additonal chiral U(1) current whose generator commutes with the superconformal generators. Its anomaly coefficient was shown to have no radiative corrections.

In the first section of this talk the supersymmetric QED model is described in the tree approximation. The action and its gauge invariance is outlined. Then the superconformal symmetries and associated currents are discussed and the super current defined. Finally the additional axial current is given. In the second section the renormalization of super QED is discussed. The normal product equations of motion are given as well as the Callan-Aymanzik equation. Then the renormalized super current is defined and its trace equation is found. The trace anomaly coefficient is seen to be β/g. Finally the renormalized axial current is defined and the Adler-Bardeen theorem is proven.

In what follows in order to simplify the technicalities it has been assumed that one could take physical matrix elements of operators as well as off-shell vacuum expectation values of time ordered products of them. Since there are massless fields in the model this in reality would be quite complicated to show and is assumed here only for convenience; References (3) and (4) contain the rigorous off-shell calculations. Also I would like to formally mention that all of the results reported here have been found in collaboration with Olivier Piguet and Klaus Sibold both at the University of Karlsruhe and are reported in References (3) and (4).

1. SUPERSYMMETRIC QED: TREE APPROXIMATION

The fields of the model are ϕ, a massless vector superfield containing the photon and photino, and $\phi_\pm, \bar{\phi}_\pm$, massive charged chiral matter fields containing charged fermi fields as well as charged scalar and pseudoscalar fields. The action is invariant under parity, charge conjugation, Poincaré, gauge and supersymmetry transformations

$$I_{inv} = -\frac{i}{4}\int dS \frac{1}{g^2} W^\alpha W_\alpha - \frac{i}{4}\int d\bar{S} \frac{1}{g^2} \bar{W}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}}$$

$$+ i\int dV[\bar{\phi}_+\phi_+ e^{g\phi} + \bar{\phi}_-\phi_- e^{-g\phi}] \qquad (1.1)$$

$$+ 4im\int dS\phi_+\phi_- + 4im\int d\bar{S}\bar{\phi}_+\bar{\phi}_-$$

where $W_\alpha = g\bar{D}\bar{D}D_\alpha\phi$. In order to quantize the model a gauge fixing term must be added to the invariant action; we add a supersymmetric Stückelberg term

$$I_g = -\frac{i}{2\alpha}\int dVDD\phi\bar{D}\bar{D}\phi \qquad . \qquad (1.2)$$

The total action is given by

$$I = I_{inv} + I_g$$

$$= \frac{i}{2}\int dV\phi 8\partial^2\phi + i\int dV[\bar{\phi}_+\phi_+ e^{g\phi} + \bar{\phi}_-\phi_- e^{-g\phi}] \qquad (1.3)$$

$$+ 4im\int dS\phi_+\phi_- + 4im\int d\bar{S}\bar{\phi}_+\bar{\phi}_-$$

where we have explicitly chosen the $\alpha = 1$ (Feynman) gauge in order to avoid the IR divergencies of the general gauge, and use of the identity $8\partial^2 = D\bar{D}\bar{D}D - 1/2\ (DD + \bar{D}\bar{D})^2$ has been made.

The structural relations needed in order to derive current (non-) conservation equations are the equations of motion

1) $\frac{\delta I}{\delta\phi(z)} = 0$ which implies

$$[D\bar{D}\bar{D}D - \frac{1}{2}(DD + \bar{D}\bar{D})^2]\phi = 8\partial^2\phi$$

$$= -g[\bar{\phi}_+\phi_+ e^{g\phi} - \bar{\phi}_-\phi_- e^{-g\phi}] \qquad (1.4)$$

2) $\frac{\delta I}{\delta\phi_+(z)} = 0$ which implies

$$\bar{D}\bar{D}[\Phi_+ e^{g\phi}] + 4im\phi_- = 0 \tag{1.5}$$

and similarly for $\phi_\pm, \Phi_-$.

Note that we can define the source of the vector field's equation of motion as the gauge current j(z)

$$J(z) \equiv [\phi_+\Phi_+ e^{g\phi} - \phi_-\Phi_- e^{-g\phi}](z) \quad . \tag{1.6}$$

The chiral matter fields' equations of motion then imply current conservation

$$\begin{aligned} &1)\ DDj = 0 \\ &2)\ \bar{D}\bar{D}j = 0 \quad . \end{aligned} \tag{1.7}$$

Applying this to $DD\frac{\delta I}{\delta\phi} = 0$ and $\bar{D}\bar{D}\frac{\delta I}{\delta\phi} = 0$ yields the gauge Ward identities which imply that the longitudinal vector field decouples from the theory

$$\begin{aligned} &1)\ \partial^2 DD\phi = 0 \\ &2)\ \partial^2 \bar{D}\bar{D}\phi = 0 \quad . \end{aligned} \tag{1.8}$$

In order to grade the conformal algebra an additional Bose generator corresponding to the (chiral) R-transformations must be added to the algebra. The generators of the super conformal symmetries are given by

<u>Bose Generators</u>

P_μ: space-time translations

$M_{\mu\nu}$: Lorentz transformations

D : dilation transformation

K_μ : conformal transformation

R : R-transformations

<u>Fermi Generators</u>

$Q_\alpha, \bar{Q}_{\dot\alpha}$: translational (Restricted) SUSY

$(\{Q_\alpha, \bar{Q}_{\dot\alpha}\} = 2\sigma^\mu_{\alpha\dot\alpha} P_\mu)$

$S_\alpha, \bar{S}_{\dot\alpha}$: conformal (special) SUSY

$(\{S_\alpha, \bar{S}_{\dot\alpha}\} = 2\sigma^\mu_{\alpha\dot\alpha} K_\mu)$.

These generators are represented by linear superspace differential operators on the superfields that is for generator G

$$[G,\phi] = -i\delta^G\phi$$

$$\begin{aligned} &1)\ \delta^Q_\alpha\phi = [\frac{\partial}{\partial\theta} + i\sigma^\mu\bar{\theta}\partial_\mu]_\alpha\phi \\ &2)\ \delta^D\phi = [d + x^\lambda\partial_\lambda + \frac{1}{2}\theta\frac{\partial}{\partial\theta} - \frac{1}{2}\bar{\theta}\frac{\partial}{\partial\bar{\theta}}]\phi \\ &3)\ \delta^R\phi = i[n + \theta\frac{\partial}{\partial\theta} + \bar{\theta}\frac{\partial}{\partial\bar{\theta}}]\phi \end{aligned} \tag{1.9}$$

etc.

where d = scale dimension of ϕ

n = R-weight of ϕ and for (anti-)(chiral)fields $n = (+\frac{2}{3}d)(-\frac{2}{3}d)$ and n = 0 for vector fields. Applying these transformations to the action we find

1) $\delta^P_\mu I = 0$, $\delta^Q_\alpha I = 0$ etc.

2) $\delta^D_\mu I \neq 0$, $\delta^K_\mu I \neq 0$, $\delta^S_\alpha I \neq 0$, $\delta^{\bar{S}}_\alpha I \neq 0$

and in particular

$$\begin{aligned}\delta^R I &= i\,\frac{8}{3}\,m\int dx[DD\phi_+\phi_- - \bar{D}\bar{D}\bar{\phi}_+\bar{\phi}_- \\ &= \int dx[DDS - \bar{D}\bar{D}\bar{S}] \quad .\end{aligned} \tag{1.10}$$

This can be written more supersymmetrically by noting that the only x-constant superfield of charges is

$$\hat{R} = R - i\theta^\alpha Q_\alpha + i\bar{\theta}_\alpha\bar{Q}^\alpha - 2\theta\sigma\mu\bar{\theta}P_\mu$$

and (1.11)

$$[Q_\alpha,\hat{R}] = -\,i\,\frac{\partial}{\partial\theta^\alpha}\,R \quad .$$

Noether's theorem tells us that all superconformal currents can be written as moments of the super current V_μ associated with the above transformation (1.11):

$$\begin{aligned}&V_\mu = \sigma^{\alpha\dot{\alpha}}_\mu\ V_{\alpha\dot{\alpha}} \qquad\qquad V_{\alpha\dot{\alpha}} = \frac{1}{2}\,\sigma^{\partial}_{\alpha\dot{\alpha}}V_\mu \\ &\partial^\mu V_\mu = i[DDS - \bar{D}\bar{D}\bar{S}] \\ &\hat{R} = \quad d^3xV_0 \\ &V_\mu \quad R_\mu, Q^\alpha_\mu, \bar{Q}^{\dot{\alpha}}_\mu, T_{\mu\nu} \quad .\end{aligned} \tag{1.12}$$

The gauge invariant supercurrent is given by

$$\begin{aligned}V_{\alpha\dot{\alpha}} = &-\frac{8i}{3}\,[D_\alpha,\bar{D}_{\dot{\alpha}}][\phi_+\phi_+e^{g\phi} + \phi_-\phi_-e^{-g\phi}] \\ &+ 8i[(D_\alpha\phi_+e^{g\phi})(\bar{D}_{\dot{\alpha}}\,{}_+e^{g\phi})e^{-g\phi} \\ &+ (D_\alpha\phi_-e^{-g\phi})(\bar{D}_\alpha\phi_-e^{-g\phi})e^{g\phi}] \\ &- \frac{4i}{3}\bar{D}\bar{D}D_\alpha\phi DD\bar{D}_{\dot{\alpha}}\phi \quad .\end{aligned} \tag{1.13}$$

Since $\partial^\mu V_\mu = -\frac{1}{2}\{D^\alpha,\bar{D}^{\dot{\alpha}}\}V_{\alpha\dot{\alpha}}$ all (non-) conservation equations follow from the generalized trace equation, $D^\alpha V_{\alpha\dot{\alpha}}$, alone. That is from the equations of motion

$$\begin{aligned}D^\alpha V_{\alpha\dot{\alpha}} &= -\,\frac{16i}{3}\,m\bar{D}_{\dot{\alpha}}\phi_+\phi_- \\ \bar{D}^{\dot{\alpha}}V_{\alpha\dot{\alpha}} &= -\,\frac{16i}{3}\,mD_\alpha\phi_+\phi_-\end{aligned} \tag{1.14}$$

which implies

$$\partial^\mu V_\mu = -\,\frac{8m}{3}\,[DD\phi_+\phi_- - \bar{D}\bar{D}\phi_+\phi_-] \quad . \tag{1.15}$$

Finally corresponding to the chiral rotation of the matter fields

$$\phi'_{\pm} = e^{i\alpha}\phi_{\pm}$$
$$\bar{\phi}'_{\pm} = e^{-i\alpha}\bar{\phi}_{\pm} \qquad (1.16)$$

is the axial current superfield J^{μ}_{5}

$$J^{\mu}_{5} \quad 4i\sigma^{\mu}_{\alpha\dot{\alpha}}[D^{\alpha},\bar{D}^{\dot{\alpha}}]I_5$$
$$I_5 = i[\bar{\phi}_+\phi_+ e^{g\phi} + \bar{\phi}_-\phi_- e^{-g\phi}] \quad . \qquad (1.17)$$

The equations of motion imply

$$1)\ DDI_S = -\,8im\bar{\phi}_+\bar{\phi}_-$$
$$2)\ \bar{D}\bar{D}I_S = -\,8im\phi_+\phi_- \qquad (1.18)$$

and the axial current non-conservation equaiton

$$\partial_{\mu}J^{\mu}_{5} = [DD,\bar{D}\bar{D}]I_5$$
$$= -\,8im[DD\bar{\phi}_+\bar{\phi}_- - \bar{D}\bar{D}\phi_+\phi_-] \quad . \qquad (1.19)$$

2. SUPERSYMMETRIC QED: RENORMALIZATION

The renormalized vacuum expectation value of time ordered products of fields is constructed through the Gell-Mann-Low expansion and the supersymmetric Bogoliubov-Parasiuk-Hepp-Zimmermann momentum space subtraction scheme (5). Intermediate normalization conditions are used so that the only counterterm is the (finite) chiral mass counterterm, "a" which fixes the pole of the chiral field propagator at m^2. The renormalized normal product equations of motion are

$$1)\quad N_3[\phi_+(z)\,\frac{\delta I}{\delta\phi_+(z)}\,] = 0 \quad \text{which implies}$$
$$N_3[\bar{D}\bar{D}\bar{\phi}_+\phi_+ e^{g\phi}] + 4(m+a)N_3[\phi_+\phi_-] = 0 \qquad (2.1)$$
$$2)\quad N_2[\phi\frac{\delta\ I}{\delta}\,] = 0 \quad \text{etc.}$$

where the N_{δ} denotes the Zimmermann normal product with δ, indicating how many momentum space subtractions to make, being greater than or equal to the dimension of the composite field. These equations have the form of the naive Euler-Lagrange equations <u>however</u> the mass terms are oversubtracted (N_3 rather than N_2)! The other structural relation needed in order to study the trace equations is the Zimmermann identity relating $N_3[\phi_+\phi_-]$ to $N_2[\phi_+\phi_-]$ this is the local Callan-Symanzik (C-S) equation

$$\sigma m N_3[\phi_+\phi_-] = \sigma m N_2[\phi_+\phi_-] - \frac{1}{8}\frac{\beta}{g}N^{*}_{3}[\,\frac{1}{g^2}W^{\alpha}W_{\alpha}]$$
$$+\frac{\gamma}{2}N_3[\bar{D}\bar{D}(\bar{\phi}_+\phi_+e^{g\phi} + \bar{\phi}_-\phi_-e^{-g\phi})] - \frac{1}{4}\,\delta N_3[\bar{D}\bar{D}\phi_+\phi_-] \qquad (2.2)$$

$\beta,\gamma,\sigma,\delta$ the Callan-Symanzik functions. Consider the one-particle irreducible ordered function of this equation with b-vector fields and $(\bar{f})f$ (anti-) al matter fields; integrate this over the chiral measure and add to the anti- al equation. This yields the global Callan-Symanzik equation

$$[m \frac{\partial}{\partial m} + \beta \frac{\partial}{\partial g} - \frac{\beta}{g} b - 2\gamma(f + \bar{f})]\Gamma^{(b,f,\bar{f})}$$

$$= 4\sigma m \Delta^{*}\Gamma^{(b,f,\bar{f})} \tag{2.3}$$

:h

$$\Delta^{*} = i\int dS N_2^{*}[\phi_+\phi_-] + i\int d\bar{S} N_2^{*}[\Phi_+\Phi_-] \quad ,$$

soft mass insertion.

ıe renormalized supercurrent is then defined by

$$V_{\alpha\dot{\alpha}} \quad V_{\alpha\dot{\alpha}}^{(tree)} - \frac{16i}{3}\,\delta\partial_{\alpha\dot{\alpha}}[\phi_+\phi_- - \Phi_+\Phi_-] \quad . \tag{2.4}$$

'he normal product equations of motion and the Zimmermann identity yield the ıupercurrent generalized trace equation

$$\bar{D}^{\dot{\alpha}} N_3[V_{\alpha\dot{\alpha}}] = -2D_\alpha S$$

$$\tag{2.5}$$

$$S = \frac{8i}{3}\sigma m N_2[\phi_+\phi_-] - \frac{i\beta}{3g}\hat{N}_3^{*}[\frac{1}{g^2} W^\alpha W_\alpha]$$

with $\hat{N}_3^{*}$ a gauge invariant normal product of WW. This implies that all superconformal anomalies go as β/g.

Finally the renormalized axial current is defined by

$$I_5 \equiv [1 + 4\gamma \frac{(m+a)}{\sigma m}]I_5^{(tree)} + 2\delta \frac{(m+a)}{\sigma m}[\phi_+\phi_- + \Phi_+\Phi_-]$$

$$\tag{2.6}$$

The normal product equations of motion and the Zimmermann identity yield the anomalous axial current non-conservation equation

$$\bar{D}\bar{D}N_2[I_5] = -8i(m + a)N_2[\phi_+\phi_-]$$

$$+ \frac{i\beta}{g}\frac{(m+a)}{\sigma m} N_2^{*}[\frac{1}{g^2} W^\alpha W_\alpha] \tag{2.7}$$

with N_2^{*} another gauge invariant normal product of WW. The anomaly coefficient is given by

$$r \quad \frac{\beta}{g}\frac{(m+a)}{\sigma m}$$

$$\tag{2.8}$$

$$= -4(m+a) < T\int dS_3 N_2[\phi_+\phi_-(3)]\tilde{\phi}(,1)\phi(0,2)^{\text{proper}} \quad .$$

In order to show that r has no radiative corrections we apply the C-S equation to r itself

$$\frac{\beta}{g}\,[g\,\frac{\partial}{\partial g} - 2]r = 4\sigma m[\Delta^* + \delta^*]r \quad . \tag{2.9}$$

where δ^* is an extra C-S function for the inserted C-S equation. In order to show that the RHS = 0 consider the axial current Ward identity for r itself, this implies

$$4\sigma m[\Delta^* + \delta^*]r = \int dV_3 N_2[Q(3)]r \tag{2.10}$$

with

$$Q = a\phi\partial^2 + b[\phi_+\Phi_+ e^{g\phi} + \phi_-\Phi_- e^{-g\phi}] \quad .$$

Then apply the gauge Ward identity and supersymmetry Ward identity to $N_2[Q(3)]r$ to show that the RHS of (2.10) equals zero. Thus

$$\beta[g\,\frac{\partial}{\partial g} - 2]r = 0 \quad . \tag{2.11}$$

Since $\beta \neq 0$ this implies that r is given by the second order triangle graphs only. (See Figure 1.)

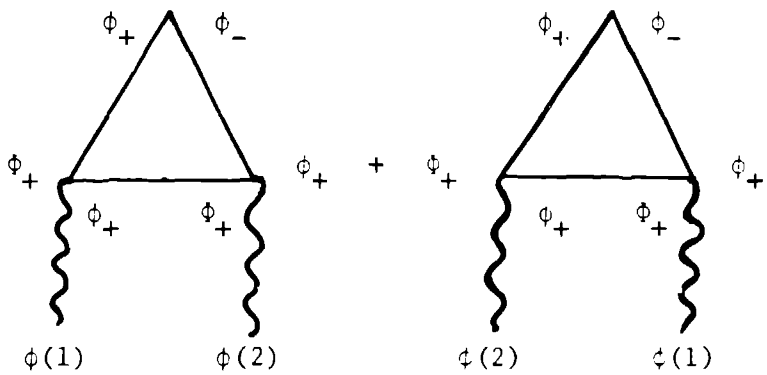

FIGURE 1: Graphical Contributions To r.

Thus

$$r = \frac{im^2 g^3}{4^5\pi^4}\int dk\,\frac{1}{[k^2-m^2+i\varepsilon]^3}$$
$$r = \frac{1}{8}\,\frac{g^2}{(16\pi)^2} \quad . \tag{2.12}$$

The divergence of the axial current is given by, recalling that $J_5 \equiv 4i\sigma^{\mu}_{\alpha\dot{\alpha}}[D^{\alpha},\bar{D}^{\dot{\alpha}}]I_5$,

$$\begin{aligned}\partial_\mu N_3[J_5^\mu] &= [DD,\bar{D}\bar{D}]N_2(I_5)\\ &= -8i(m + a)[DDN_2[\phi_+\phi_-] - \bar{D}\bar{D}N_2[\Phi_+\Phi_-]]\\ &\quad + \frac{ig^2}{8(16\pi)^2}\left\{DDN_2^*[\frac{1}{g^2}W^\alpha W_\alpha]\right.\\ &\quad \left. - \bar{D}\bar{D}N_2^*[\frac{1}{g^2}\bar{W}_{\dot{\alpha}}\bar{W}^{\dot{\alpha}}]\right\} \quad .\end{aligned} \tag{2.13}$$

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